PERIOD–V

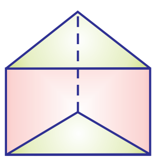
M12CH19

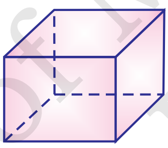
TOPIC
# 19 Solid Geometry

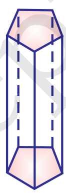
## 19.1. SURFACE AREA OF A PRISM

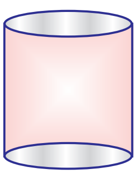
Prisms are solid with uniform cross-section. The cross-section may be triangular, rectangular, circular of shapes. A prism is named by the spade of its cross-section.

For example:


Triangular prism


Rectangular prism


Pentagonal prism


Cylinder

## 19.2. CUBOID

It is a prism with rectangular faces. All the sides of a cuboid are rectangles.

Formula used to calculate surface area of a cuboid is:

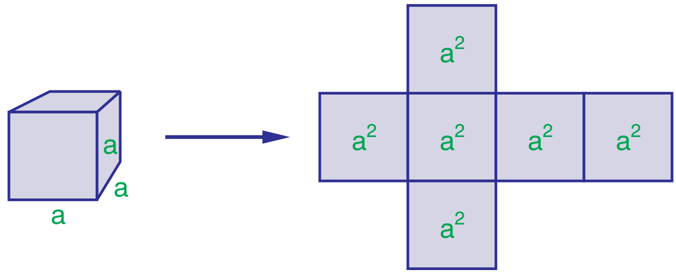
Surface area of a cuboid = $2(lb + bh + hl)$

where $l$, $b$, and $h$ are respectively the three edges of the cuboid.

## 19.3. CUBE

Recall that a cuboid having equal length, equal length, breadth and height is called a cube. If each edge of the cube is '$a$', the surface area of this cube would be

$2(a \times a + a \times a + a \times a)$ *i.e.*, (see figure)




Formula used to calculate surface area of cuboid is

Surface area of a cube = $6a^2$

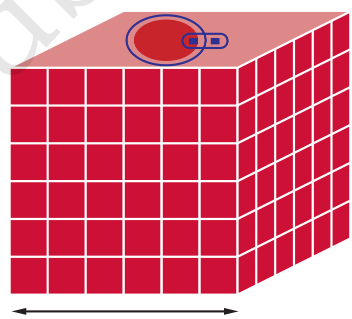
**Example 1.** *The length, breadth and height of a cuboid are 15 cm, 10 cm and 20 cm respectively. Find the surface area.*

**Solution.** Surface area of a cuboid = $2(lb + bh + hl)$

So, Surface area = $2[(15 \times 10) + (10 \times 20) + (20 \times 15)]$ cm$^2$

= $2(150 \times 200 + 300)$cm$^2$ = $2 \times 650$ cm$^2$ = **1,300 cm$^2$**

**Example 2.** *Runesha has built a cubical water tank with lid for his house, with each outer edge 1.5 m long. He gets the outer walls of the tank covered with square tiles of side 25 cm (see fig.) Find how much he will spend for the tiles, if the cost of the tiles is 3600 L$ per dozen.*




**Solution.** Runesha is getting the four outer walls of the tank covered with tiles. He needs to calculate the surface area of the walls, to know the number of tiles required.

Edge of the cubical tank = 1.5 m = 150 cm

So, Surface area of the tank = $4 \times 150 \times 150$ cm$^2$

Area of each square tile = Side × Side = $25 \times 25$ cm$^2$

So, the number of tiles required

$$= \frac{\text{Surface area of the tank}}{\text{Area of each tile}} = \frac{4 \times 150 \times 150}{25 \times 25} = 144$$

Cost of 1 dozen tiles, i.e., 12 tiles = 3,600 L$

Therefore, the cost of one tile = $\frac{3,600}{12}$ L$ = 300 L$

So, the cost of 144 tiles = 144 × 300 L$ = 43,200 L$

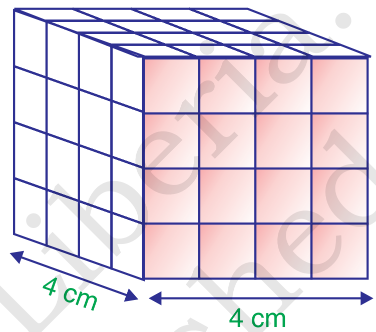
## 19.4. VOLUME OF A CUBOID

Solid objects occupy space. The measure of this occupied space is called the Volume of the object.

Formula used to calculate volume of a cuboid

$$= \text{length} \times \text{breadth} \times \text{height}$$

$$\Rightarrow \quad V = l \times b \times h$$




Now, consider figure it is a cube.

The cube is made of a number of unit cubes each of side 1 cm.

Let us count the number of unit cubes used in making this cube. It is found to be 64.

Volume of 1 unit cube = 1 $cm^3$

Volume of 64 unit cubes = 64 $cm^3$ = 4 cm × 4 cm × 4 cm = $(4\text{ cm})^3$

Formula used to calculate

Hence, volume of a cube = $(\text{side})^3$

$$\Rightarrow \quad V = a^3$$

**Example 3.** *What is the volume of a rectangular solid with edge 9 m, 3.5 m and 2.8 m?*

**Solution.** Volume = $l \times b \times h$ = 9 m × 3.5 m × 2.8 m = **88.2 $m^3$**

**Example 4.** *A rectangular box has a volume of 9 $m^3$ and a base 50 cm by 25 cm. Find its height.*

**Solution.** Dimensions of the box are:

Volume (V) = 9 $m^3$; length (l) = 50 cm = 0.5 m; breadth (b) = 25 cm = 0.25 m

Since, volume = $l \times b \times h$

So, $$\text{height } (h) = \frac{\text{Volume}}{l \times b}$$

$$h = \frac{9\text{ m}^3}{0.5\text{ m} \times 0.25\text{ m}} = 72\text{ m}$$

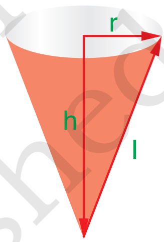
$$\therefore \quad h = 72\text{ m}$$

## 19.5. SURFACE AREA OF A RIGHT CIRCULAR CONE

Formula used to calculate curved surface area of a cone is

$$\text{Curved surface area of cone} = \frac{1}{2} \times l \times 2\pi r = \pi rl$$

where $r$ is its base radius and $l$ is its slant height.




Now, suppose h is the height of the cone. Then using Pythagoras theorem we have, $l^2 = r^2 + h^2$.

Therefore, $l = \sqrt{r^2 + h^2}$

If the base of the cone is closed, then a circular piece of paper of radius $r$ is also required whose area is $\pi r^2$.

So, Formula used to calculate Total surface area of a cone is

$$\text{Total surface area of cone} = \pi rl + \pi r^2 = \pi r(l + r)$$

**Example 5.** *The slant height of a cone is 10 cm and the base radius is 7 cm. Find the curved surface area.*

**Solution.** Curved surface area $= \pi rl = \frac{22}{7} \times 7 \times 10 \text{ cm}^2 = \mathbf{220\ cm^2}$

## 19.6. VOLUME OF A RIGHT CIRCULAR CONE

We observe that, three times the volume of the cone makes up the volume of a cylinder. Also, the cylinder has the same base radius and the same height as the cone has.

It means the volume of the cone = one-third the volume of the cylinder.

Formula used to calculate volume of a cone

So, $\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$

where $r$ is the base radius and $h$ is the height of the cone.

**Example 6.** *The height and the slant height of a cone are 21 cm and 28 cm respectively. Find the volume of the cone.*

**Solution.** By Pythagorean theorem $l^2 = r^2 + h^2$

$$\Rightarrow \quad r = \sqrt{l^2 - h^2} = \sqrt{28^2 - 21^2} \text{ cm} = 7\sqrt{7} \text{ cm}$$

So, volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 7\sqrt{7} \times 7\sqrt{7} \times 21 \text{ cm}^3$$

$$= \mathbf{7,546 \text{ cm}^3}$$

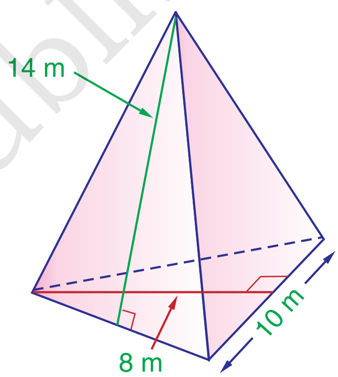
## 19.7. SURFACE AREA OF A PYRAMID

A pyramid has sides that are triangular-faced and a base. The base can be of any shape.

Formula used to calculate

$$S = l^2 + 2lh$$

**Example 7.** *Find the surface area of the regular pyramid shown in figure.*




**Solution.** To help visualize the surface area clearly, sketch a net. Then use the net to find the area of base and the area of each lateral face.

Area of base,

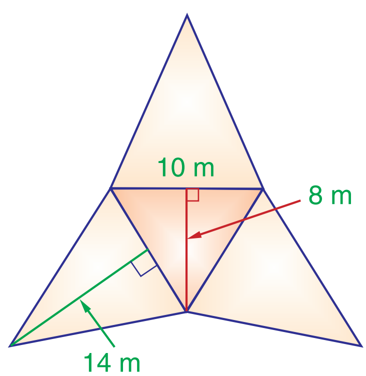
$$A = \frac{1}{2} \times 10 \times 8$$

$$= 40 \text{ m}^2$$

Area of each lateral face,

$$A = \frac{1}{2} \times 10 \times 14$$

$$= 70 \text{ m}^2$$




Finally, the surface area of the regular pyramid,

$$S = \text{area of base} + \text{areas of lateral faces}$$

$$= 40 + \underbrace{70 + 70 + 70}$$

$$= \mathbf{250 \text{ m}^2}$$

The pyramid has three congruent lateral faces. Count the area times.

The surface area is **250 square metre.**

## 19.8. VOLUME OF PYRAMID

Formula used to calculate volume of the pyramid

$$\text{Volume of the pyramid} = \frac{1}{3}\text{area of the base} \times \text{height}$$

## 19.9. SURFACE AREA OF A CYLINDER

This is the formula for the total surface area of a given cylinder whose radius is $r$ and height is $h$

Total surface area of a cylinder (TSA) = $2\pi(h + r)$

## 19.10. VOLUME OF CYLINDER

Formula used to calculate

Volume of a cylinder = $\pi r^2 h$ cubic units

**Example 8.** *Find the total surface area of a container in a cylindrical shape whose diameter is 28 cm and the height is 15 cm.*

**Solution.** Given, diameter = 28 cm, so radius = 28/2 = 14 cm and height = 15 cm

By the formula of total surface are, we know

$$TSA = 2\pi r(h + r) = 2 \times (22/7) \times 14 \times (15 + 14)$$

$$TSA = 2 \times 22 \times 2 \times 29$$

$$TSA = 2552 \text{ sq. cm}$$

Hence, the total surface area of the container is 2552 sq. cm.

## 19.11. SURFACE AREA OF A SPHERE

A sphere is a three dimensional circular figure. All points on its surface are equidistant from its centre.

Formula used to calculate surface area of a sphere is

Surface area of a sphere = $4\pi r^2$

where $r$ is the radius of the sphere.

**Example 9.** *Find the surface area of a sphere of radius 7 cm.*

**Solution.** The surface area of a sphere of radius 7 cm would be

$$4\pi r^2 = 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \mathbf{616\ cm^2}$$

Formula used to calculate the volume of a sphere is

Volume of a sphere = $\frac{4}{3}\pi r^3$

where $r$ is the radius of the sphere.

**Example 10.** *Find the volume of a sphere of radius 11.2 cm.*

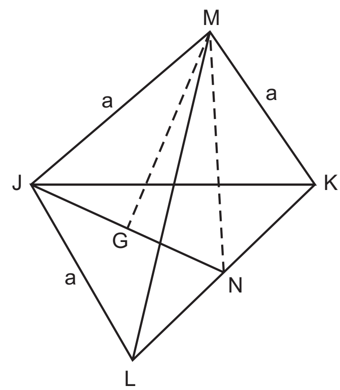
**Solution.** Required volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 11.2 \times 11.2 \times 11.2 \text{ cm}^3$

$= \mathbf{5,887.32\ cm^3}$

## 19.12. TETRAHEDRON

A pyramid on a triangular base is called a **tetrahedron**. In other words, a tetrahedron is a solid bounded by four triangular faces. Evidently a tetrahedron is a triangular pyramid. If the base of a tetrahedron is an equilateral triangle and the other triangular faces are isosceles triangles then it is called a **right tetrahedron**. A tetrahedron is said to be regular when all its four faces are equilateral triangles. Clearly, these equilateral triangles are congruent to one another.

A regular tetrahedron has been shown in the given figure. M is the vertex and the equilateral triangle JLK is the base of the regular tetrahedron. JL, LK, KJ, MJ, ML and MK are its six edges and three lateral faces are congruent equilateral triangles LKM, KJM and JLM. If G be the centroid of the base JLK and N, the mid-point of the side LK then MG is the height and MN, the slant height of the regular tetrahedron.




Let $a$ be the length of an edge of a regular tetrahedron. Then,

1. Area of the slant height surface of the regular tetrahedron
   = sum of the areas of three congruent equilateral triangles
   $= 3 \cdot (\sqrt{3}) / 4a^2$ square units.
2. Area of the whole surface of the regular tetrahedron
   = sum of the areas of four congruent equilateral triangles
   $= 4 \cdot (\sqrt{3}) / 4a^2$ square units.
   $= \sqrt{3}a^2$ square units;

3. Volume of the regular tetrahedron

= 1/3 × area of the base × height

$= (1/3) \cdot (\sqrt{3})/4 \times a^2 \times (\sqrt{2})(\sqrt{3})a$ square units.

$= (\sqrt{2}/12)a^3$ cubic units.

**Worked-out Problems in finding surface area and volume of a tetrahedron**

**Example 11.** *Each edge of a regular tetrahedron is of length 6 metre. Find its total surface area and volume.*

**Solution.** A regular tetrahedron is bounded by four congruent equilateral triangles.

By question, each edge of the tetrahedron is of length 6 metre

Therefore, the total surface area of the tetrahedron

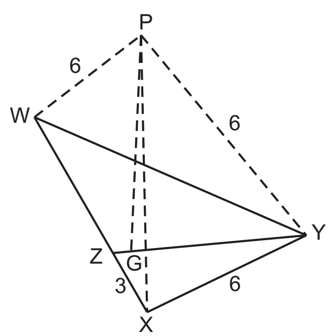
= 4 × area of the equilateral triangle of side 6 metres

$= 4 \times (\sqrt{3})/4 \cdot 6^2$ square metre

$= 36\sqrt{3}$ square metere

Therefore, from the right-angled ΔXYZ we get;

$YZ^2 = XY^2 - XZ^2 = 6^2 - 3^2$




[Since, $\overline{XY}$ = 6 m (given) and

$\overline{XZ} = 1/2 \cdot \overline{WX}$ =3 m]

or $YZ^2 = 27$ or $\overline{YZ} = 3\sqrt{3}$

Let G be the centroid of the triangle WXY. Then,

$\overline{YG} = 2/3 \cdot \overline{YZ} = 2/3 \cdot 3\sqrt{3}$

Let $\overline{PG} \perp \overline{YG}$, hence from Δ PYG we get

$PG^2 = PY^2 - YG^2 = 6^2 - (2\sqrt{3})^2$ [Since $\overline{PY}$ = 6m]

or $PG^2 = 36 - 12 = 24$ or $\overline{PG} = 2\sqrt{6}$

Therefore, the required volume of the tetrahedron

= 1/3 × (area of ΔWXY) × $\overline{PG}$

$= 1/3 \cdot (\sqrt{3})/4 \cdot 6^2 \cdot 2\sqrt{6}$ cubic metre

$= 18\sqrt{2}$ cubic metre

## 19.13. WHAT IS THE SURFACE AREA OF A HEXAGONAL PRISM

The surface are of a hexagonal prism is defined as the total region covered by the surfaces of a hexagonal prism. Since, it has flat base, this it has a total surface area as well as a curved/lateral surface area. A hexagonal prism has 8 faces, 18 edges, and 12 vertices. It has equal top and bottom bases with diagonals crossing the center point of a regular hexagon.

The surface area of a hexagonal prism is expressed in square units. common units being square meters, square centimeters, square inches, etc. Just like other three-dimensional shapes, a hexagonal prism can also have two types of areas,

- Total Surface Area (TSA)
- Lateral Surface Area (LSA)

## 19.14. FORMULA OF SURFACE AREA OF HEXAGONAL PRISM

Total Surface Areas, TSA = 2(area of hexagon base) + 6(area of triangle face) sq. units = $6b(a + h)$ or $6ah + 3\sqrt{3}a^2$ (in case of regular hexagonal prism)

Lateral Surface Area, LSA = $Ph$ = 6(area of the rectangle)

= $6ah$ sq. units.

where $a$ = base length, $a$ = apothem length, $h$ = height

## 19.15. LATERAL SURFACE AREA OF A HEXAGONAL PRISM

The lateral surface area of the hexagonal prism is the sum of the area of 6 rectangular faces. Therefore, the lateral surface area, L = $6ah$ = $6ah$ aq.

## 19.16. TOTAL SURFACE AREA OF A HEXAGONAL PRISM

Total Surface Area, T = 6(area of rectangle face) + 2(area of hexagon base) sq. units = $6ah + 3\sqrt{3}a^2$

**Example 12.** *Determine the total surface area and the lateral surface area of a hexagonal prism with a base length of 4 inches and height of 11 inches.*

**Solution.** Given $a$ = 4 inches and $h$ = 11 inches

Lateral Surface Area of Hexagonal Prism = $6ah$ = 6 × 4 × 11 = 264 square inches

Total Surface Area of Hexagonal Prism

$$= \text{LSA} + 3\sqrt{3}a^2 = 264 + 3\sqrt{3} \times (4)^2$$

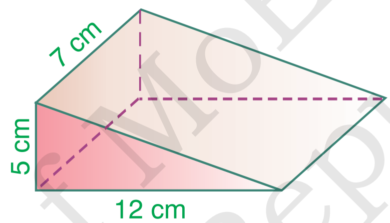
$\Rightarrow$ TSA = 264 + 83.136

$\Rightarrow$ TSA = 347.136 square inches

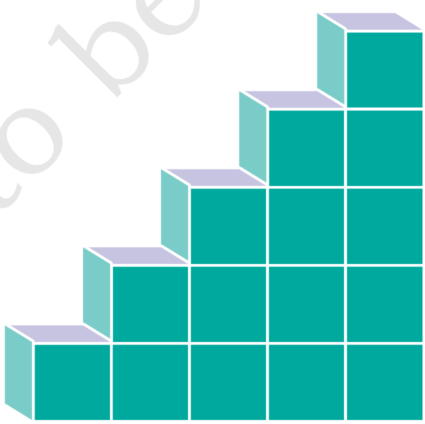
Therefore, the total surface area of the hexagonal prism is 347.136 square inches.

## EXERCISE

**1.** What is the total surface area of this prism?

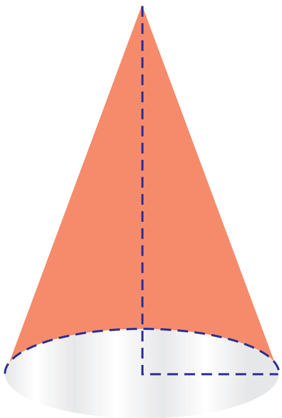



**2.** A child is playing with building blocks, which are of the shape of cubes. It has built a structure as shown in figure. The edge of each cube is 3 cm. Find the volume of the structure built by the child.

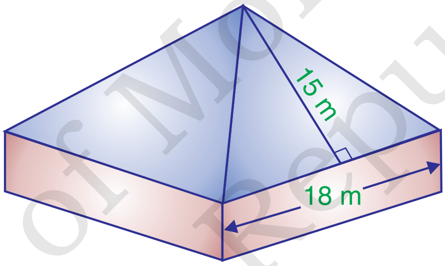


**3.** The height of a cone is 16 cm and its base radius is 12 cm. Find the curved surface area and the total surface area of the cone. (Use $\pi$ = 3.14).

**4.** Find the volume of a cone (see figure) with a radius of 7 cm and a height of 30 cm.



**5.** The roof is shaped like a square pyramid. One bundle of shingles covers 20 square metre. How many bundles should you buy to cover the roof?




**6.** Find the total surface area of a hexagonal prism with the base of edge as 6 units and height as 12 units.

**7.** Find the height of the hexagonal prism if its total surface area is 396 sq feet, apothem length is 3 feet, base length is 6 feet.